

Lec 9

state space

→ The system is described its dynamics in time domain

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) \\ y(t) &= C x(t) + D u(t) \end{aligned}$$

Two state equation

Where $u(t) \rightarrow$ i/p

$y(t) \rightarrow$ o/p

$(D=0) \rightarrow$ (Physical system) جاء في T.F جاء في 1

$n \rightarrow$ system order

$A_{n \times n} \Rightarrow$ system matrix

$B_{n \times 1} \Rightarrow$ i/p matrix

$C_{1 \times n} \Rightarrow$ o/p matrix ($x(t) \Rightarrow$ state vector $n \times 1$)

$$x(t)_{n \times 1} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

(measuring values)

$x_1(t), x_2(t), \dots, x_n(t) \rightarrow$ states of the system.

Control Lec

~~state space~~
system

T-F
(differential
eqns)

Block diagram
or signal
flow graph

two state
eqn.

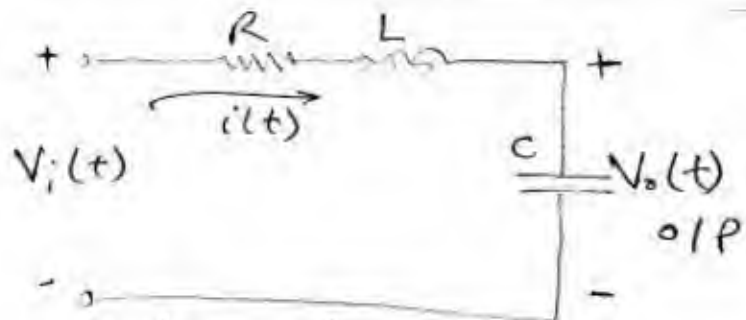
Physical
model

$$\begin{cases} \dot{x} = Ax + Bu \\ y = cx + Du \end{cases}$$

*

$$i(t) = x(t)$$

$$V_o(t) = x_2(t)$$



$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \\ \end{pmatrix} V_i(t)$$

$$y(t) = \begin{pmatrix} \\ \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

$$V_i(t) = V_R + V_L + V_o$$

$$= \underbrace{i(t)}_{x_1(t)} R + L \underbrace{\frac{di(t)}{dt}}_{\dot{x}_1(t)} + \underbrace{V_o(t)}_{x_2(t)}$$

$$L \dot{x}_1(t) = -R x_1(t) - x_2(t) + V_i(t)$$

$$\dot{x}_1(t) = -\frac{R}{L} x_1(t) - \frac{1}{L} x_2(t) + \frac{1}{L} V_i(t)$$

$$\underbrace{i(t)}_{x_1(t)} = C \underbrace{\frac{dV_o(t)}{dt}}_{\dot{x}_2(t)}$$

$$\dot{x}_2(t) = \frac{1}{C} x_1(t)$$

$$V_o(t) = x_2(t) \Rightarrow y(t) = x_2(t)$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{L} & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} V_i(t)$$

$$y(t) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

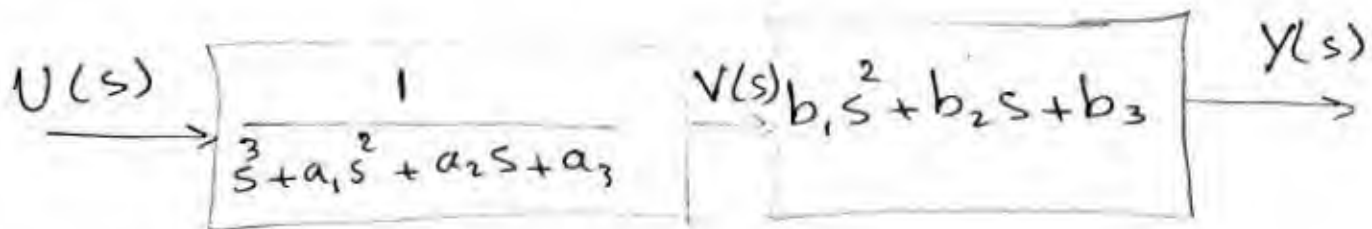
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Canonical Forms for state space

1 Controllable Form

3rd order system

$$T.F = \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$



assume

$$\dot{V} = X_1 \quad (X_1(t) = X_2(t) \rightarrow 1)$$

$$\dot{X}_2 = X_3(t) \rightarrow 2$$

$$\dot{X}_3 = \dots \downarrow$$

$$\frac{\cancel{V(s)}}{U(s)} \rightarrow \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$(s^3 + a_1 s^2 + a_2 s + a_3) V(s) = U(s) \downarrow \mathcal{L}^{-1} \cdot T$$

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$$\underbrace{\ddot{V}_1(t)}_{\dot{x}_3} + a_1 \underbrace{\ddot{V}(t)}_{\dot{x}_3} + a_2 \underbrace{\dot{V}(t)}_{x_2} + a_3 \underbrace{V(t)}_{x_1} = u(t)$$

~~$\ddot{x}_3 + a_1$~~

$$\dot{x}_3 = -a_3 x_1 - a_2 x_2(t) - a_1 x_3 + u(t) \quad \rightarrow \textcircled{3}$$

~~1, 2, 3~~ \rightarrow first state eqn.

$$\frac{Y(s)}{V(s)} = b_1 s^2 + b_2 s + b_3$$

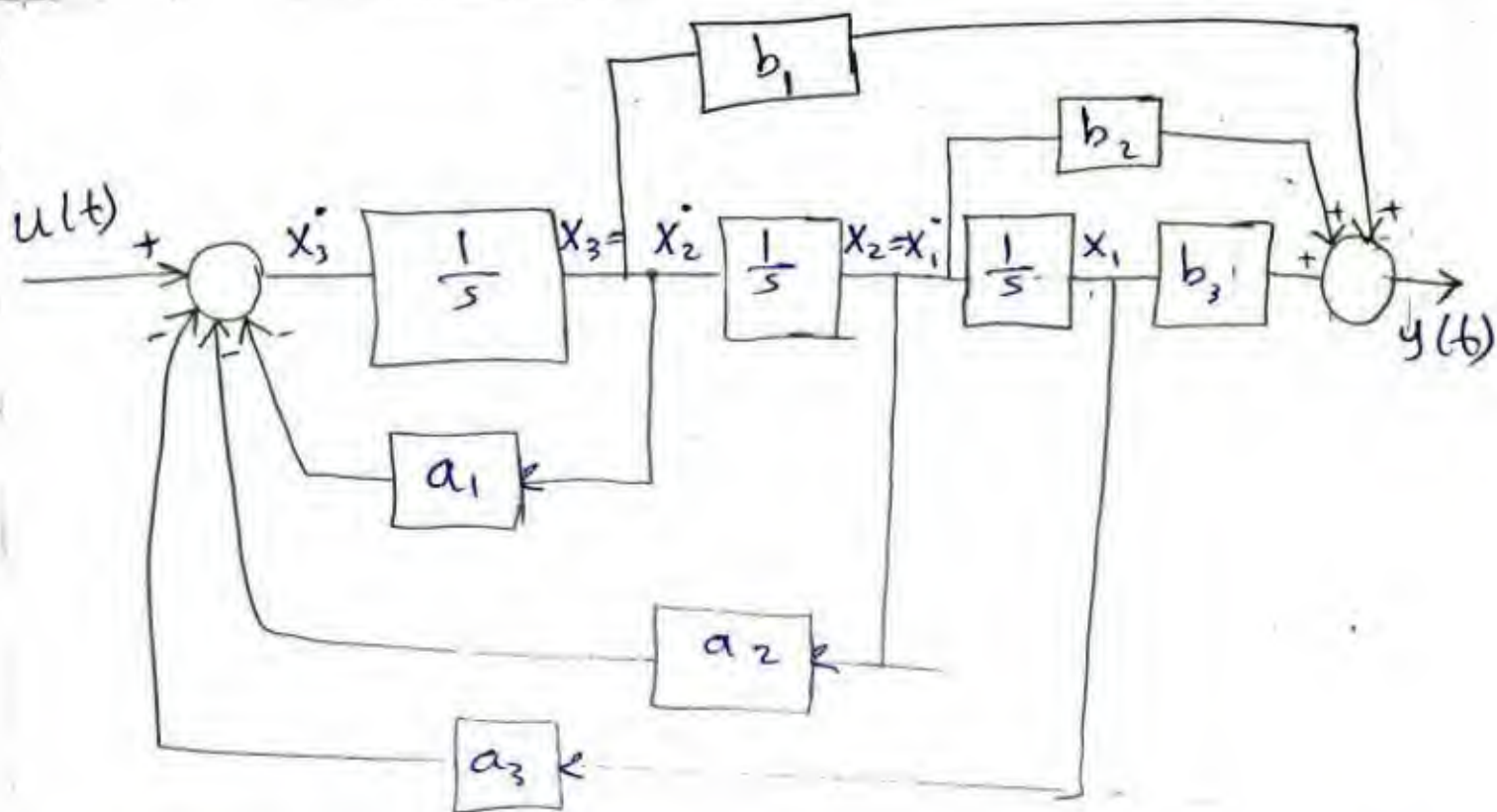
$$\mathcal{L}^{-1} \Downarrow \quad Y(s) = (b_1 s^2 + b_2 s + b_3) V(s)$$

$$y(t) = b_1 \underbrace{\ddot{V}(t)}_{\dot{x}_3} + b_2 \underbrace{\dot{V}(t)}_{x_2} + b_3 \underbrace{V(t)}_{x_1}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = (b_3 \quad b_2 \quad b_1) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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معطى النظام قابل للتحكم (Controllable)
 (state) معطى النظام قابل للتحكم (Controllable)

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

First \rightarrow check that Coeff. of $s^3 = 1$
 أكبر أس

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

معاملات النظام
 معطى

$$y(t) = \underbrace{(b_3 \quad b_2 \quad b_1)}_{\text{معاملات البسط معكوسة}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

لكن بنفس الإشارة.

$$T.F = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = (b_4 \quad b_3 \quad b_2 \quad b_1) x(t)$$

2nd order $T.F = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = (b_2 \quad b_1) x(t)$$

[2] observable form

3rd order system

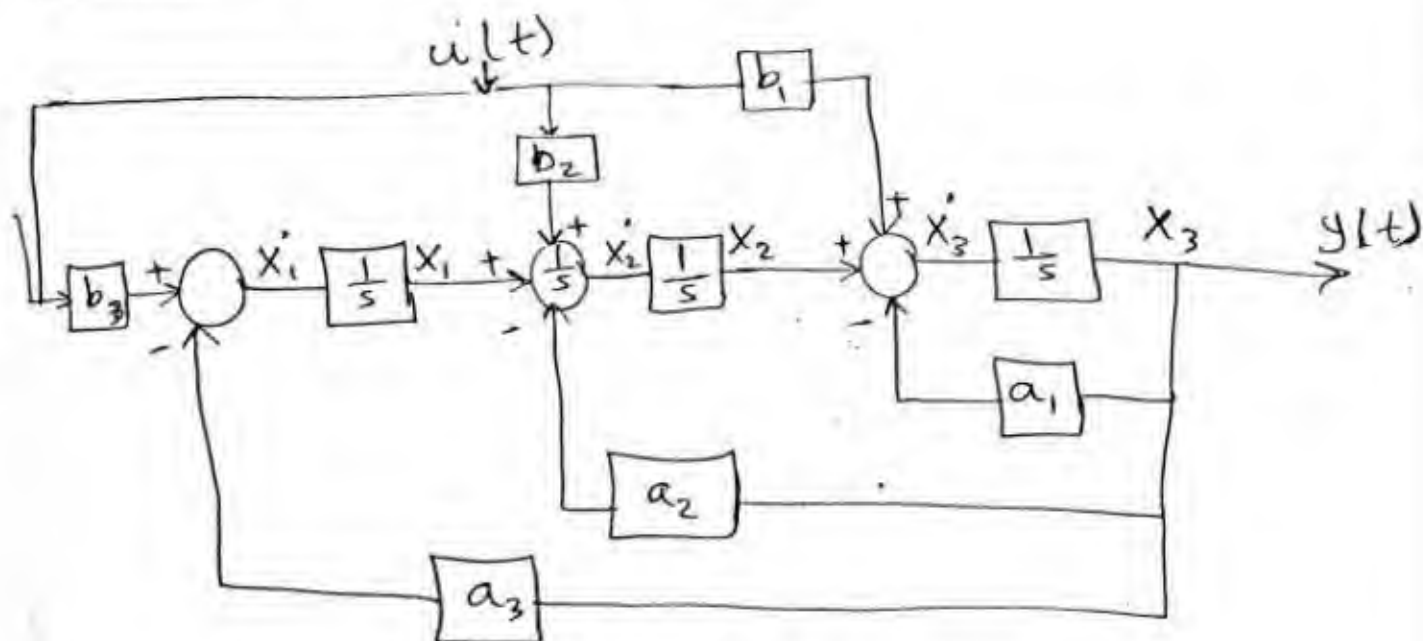
$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\left\{ \begin{array}{l} A_o = A_c^T \\ B_o = B_c^T \\ C_o = C_c^T \end{array} \right.$$

$$\dot{X}(t) = \begin{pmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{pmatrix} X(t) + \begin{pmatrix} b_3 \\ b_2 \\ b_1 \end{pmatrix} u(t)$$

$$y(t) = (0 \quad 0 \quad 1) X(t)$$

~~Block diagram of the system~~



4th order system

$$T.F. = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

$$\dot{X}(t) = \begin{bmatrix} 0 & 0 & 0 & -a_4 \\ 1 & 0 & 0 & -a_3 \\ 0 & 1 & 0 & -a_2 \\ 0 & 0 & 1 & -a_1 \end{bmatrix} X(t) + \begin{bmatrix} b_4 \\ b_3 \\ b_2 \\ b_1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} X(t) \quad \text{مخرج النظام}$$

$\rightarrow x_1, x_2, x_3, x_4$

[EX]

$$T.F. = \frac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)}$$

* Find the state-space model in Controllable and observable forms.

* Draw the state diagram for each case

[9]

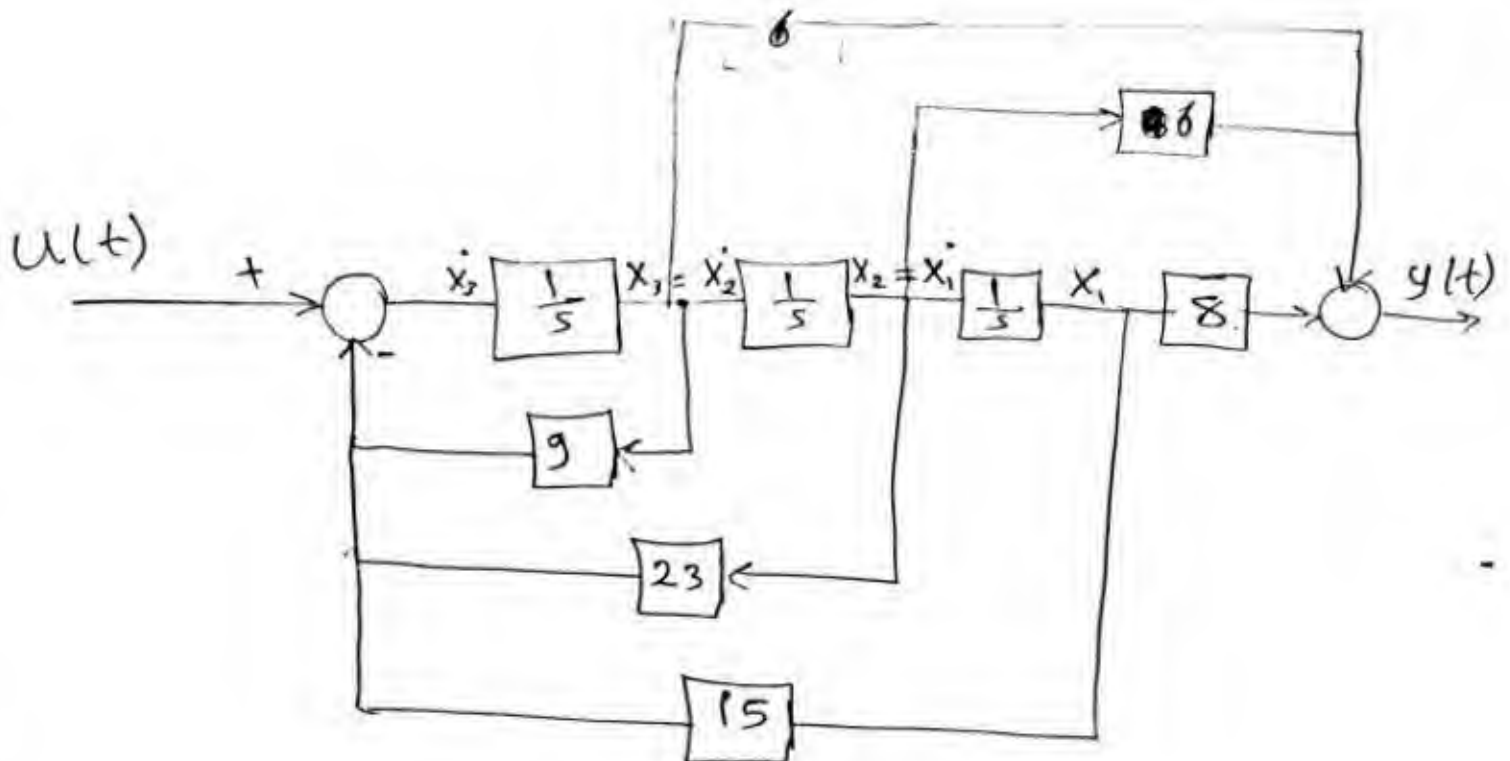
Sol

$$T.F = \frac{s^2 + 6s + 8}{s^3 + 9s^2 + 23s + 15}$$

[1] Controllable Form

$$\dot{X}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -23 & -9 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = (8 \quad 6 \quad 1) X(t)$$

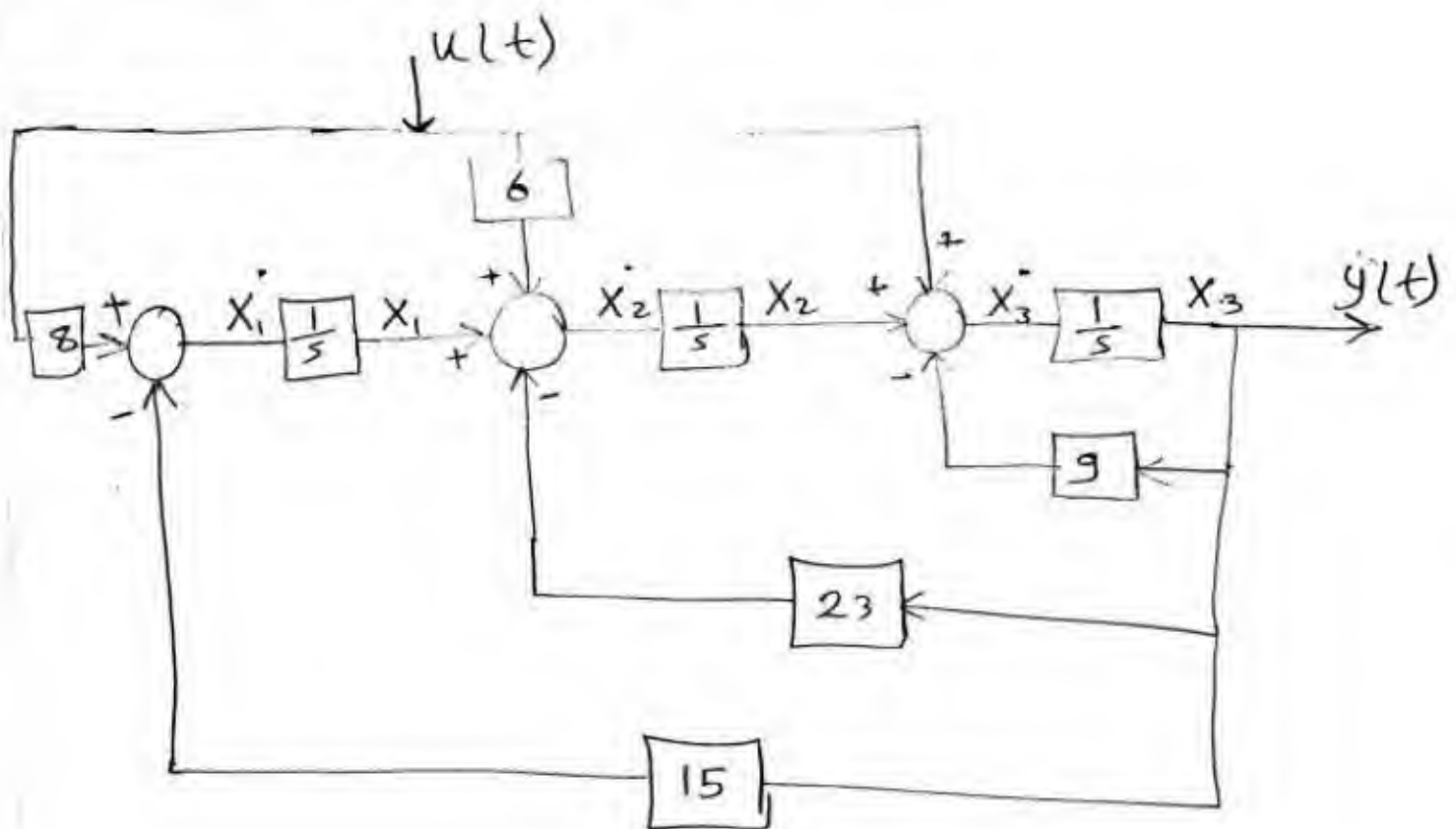


[10]

2] observable

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & -15 \\ 1 & 0 & -23 \\ 0 & 1 & -9 \end{bmatrix} x(t) + \begin{pmatrix} 8 \\ 6 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t)$$



IV

[3] Diagonal Form

3rd order system,

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{(s+p_1)(s+p_2)(s+p_3)} = \frac{A_1}{(s+p_1)} + \frac{A_2}{(s+p_2)} + \frac{A_3}{(s+p_3)}$$

$$Y(s) = \underbrace{\frac{U(s) \cdot A_1}{(s+p_1)}}_{X_1(s)} + \underbrace{\frac{U(s) \cdot A_2}{(s+p_2)}}_{X_2(s)} + \underbrace{\frac{U(s) \cdot A_3}{(s+p_3)}}_{X_3(s)}$$

$$X_1(s) = \frac{U(s)}{s+p_1} \Rightarrow U(s) = (s+p_1) X_1(s)$$

$$u(t) = \dot{x}_1(t) + p_1 x_1(t)$$

$$\boxed{\dot{x}_1 = -p_1 x_1 + u(t)}$$

$$\dot{X}_2(t) = -p_2 X_2 + u(t)$$

$$\dot{X}_3(t) = -p_3 X_3 + u(t)$$

$$\dot{X}(t) = \begin{bmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & 0 \\ 0 & 0 & -p_3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

$$Y(s) = A_1 X_1(s) + A_2 X_2(s) + A_3 X_3(s)$$

$$y(t) = A_1 x_1(t) + A_2 x_2(t) + A_3 x_3(t)$$

$$y(t) = (A_1 \quad A_2 \quad A_3) X(t)$$

$$T.F = \frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$= \frac{b_1 s^2 + b_2 s + b_3}{(s+p_1)(s+p_2)(s+p_3)} = \frac{A_1}{s+p_1} + \frac{A_2}{s+p_2} + \frac{A_3}{s+p_3}$$

3 Poles $\Rightarrow -p_1, -p_2, -p_3$.

$$\dot{X}(t) = \begin{pmatrix} -p_1 & 0 & 0 \\ 0 & -p_2 & 0 \\ 0 & 0 & -p_3 \end{pmatrix} X(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t)$$

حل عناصرها
تتكرر 1

ليه القطر الرئيس عبارة عن Poles

$$y(t) = (A_1 \quad A_2 \quad A_3) X(t)$$

عناصر البسط

for repeated Poles

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{(s + p_1)^2 (s + p_2)}$$

Poles $\Rightarrow -p_1, -p_1, -p_2$

$$Y(s) = \frac{U(s) A_1}{(s + p_1)^2} + \frac{U(s) A_2}{s + p_1} + \frac{U(s) A_3}{s + p_2}$$

$$X_1(s) = \frac{U(s)}{(s + p_1)^2} = \frac{U(s)}{s + p_1} \cdot \frac{1}{s + p_1}$$

$$X_1(s) = \frac{X_2(s)}{s+p_1} \rightarrow \boxed{\dot{x}_1 = -p_1 x_1 + x_2}$$

$$X_2(s) = \frac{U(s)}{s+p_1} \rightarrow \dot{x}_2 = -p_1 x_2 + u(t)$$

$$\dot{x}_3 = -p_2 x_3 + u(t)$$

$$\dot{X}(t) = \begin{pmatrix} -p_1 & 1 & 0 \\ 0 & -p_1 & 0 \\ 0 & 0 & -p_2 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$\text{Ex: T.f} = \frac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)}$$

- find the state space mode in Diagonal form

- Draw the state diagram

$$\text{T.f} = \frac{s^2 + 6s + 8}{(s+1)(s+3)(s+5)} \quad \text{Sol}$$

$$\text{using P.f}$$

$$= \frac{A_1}{(s+1)} + \frac{A_2}{(s+3)} + \frac{A_3}{(s+5)}$$

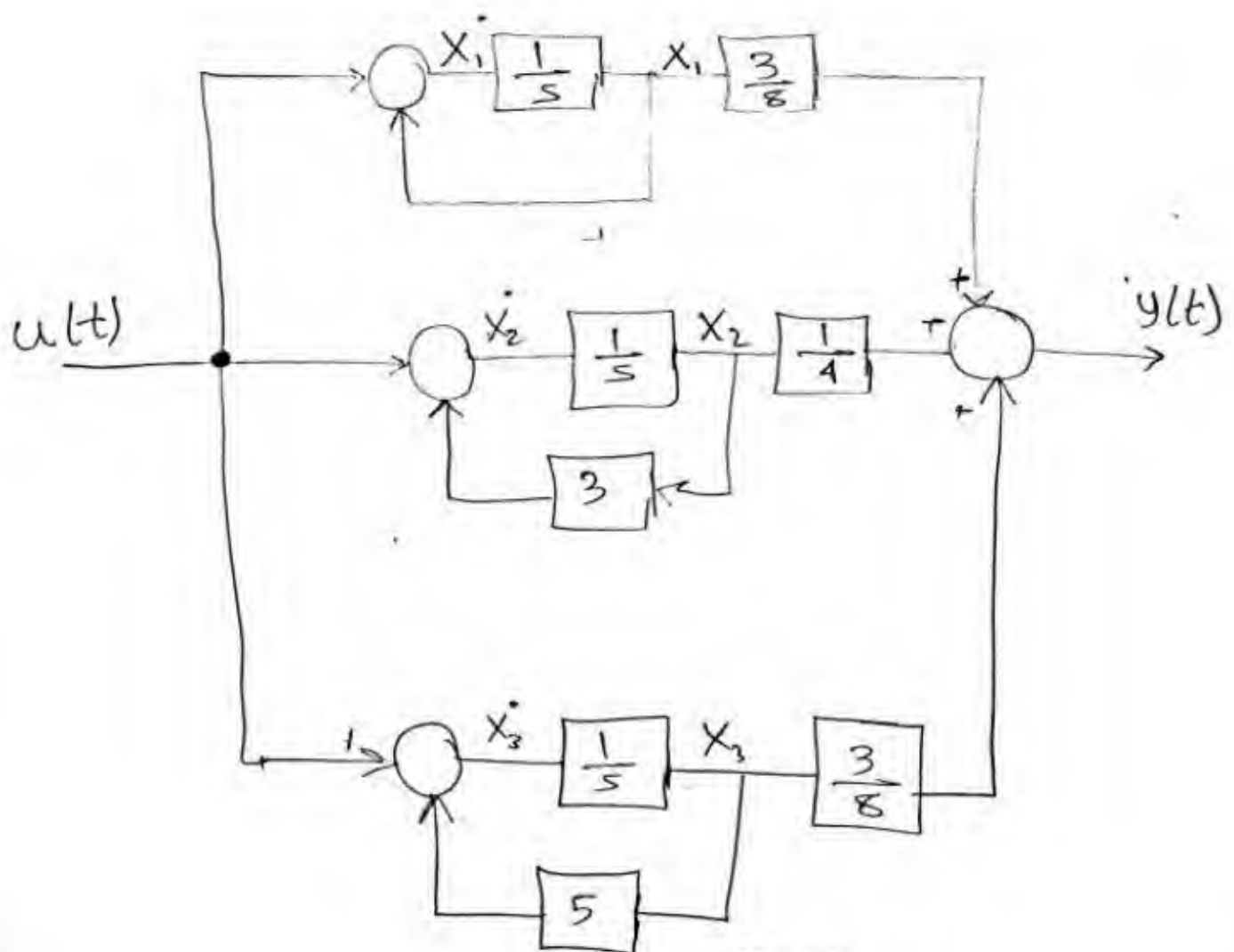
$$A_1 = \frac{1-6+8}{(2)(4)} = \frac{3}{8} \quad \left| \quad A_2 = \frac{9-18+8}{(-2)(2)} = \frac{-1}{-4} = \frac{1}{4} \right.$$

$$A_3 = \frac{25-30+8}{(-4)(-2)} = \frac{3}{8}$$

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$$\dot{x}(t) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} \frac{3}{8} & \frac{1}{4} & \frac{3}{8} \end{pmatrix} x(t)$$



Ex T.F = $\frac{2s^2 + 6s + 5}{(s+1)^2 (s+2)} \Downarrow \text{p.f}$

$$T.F = \frac{A_1}{(s+1)^2} + \frac{A_2}{s+1} + \frac{A_3}{s+2}$$

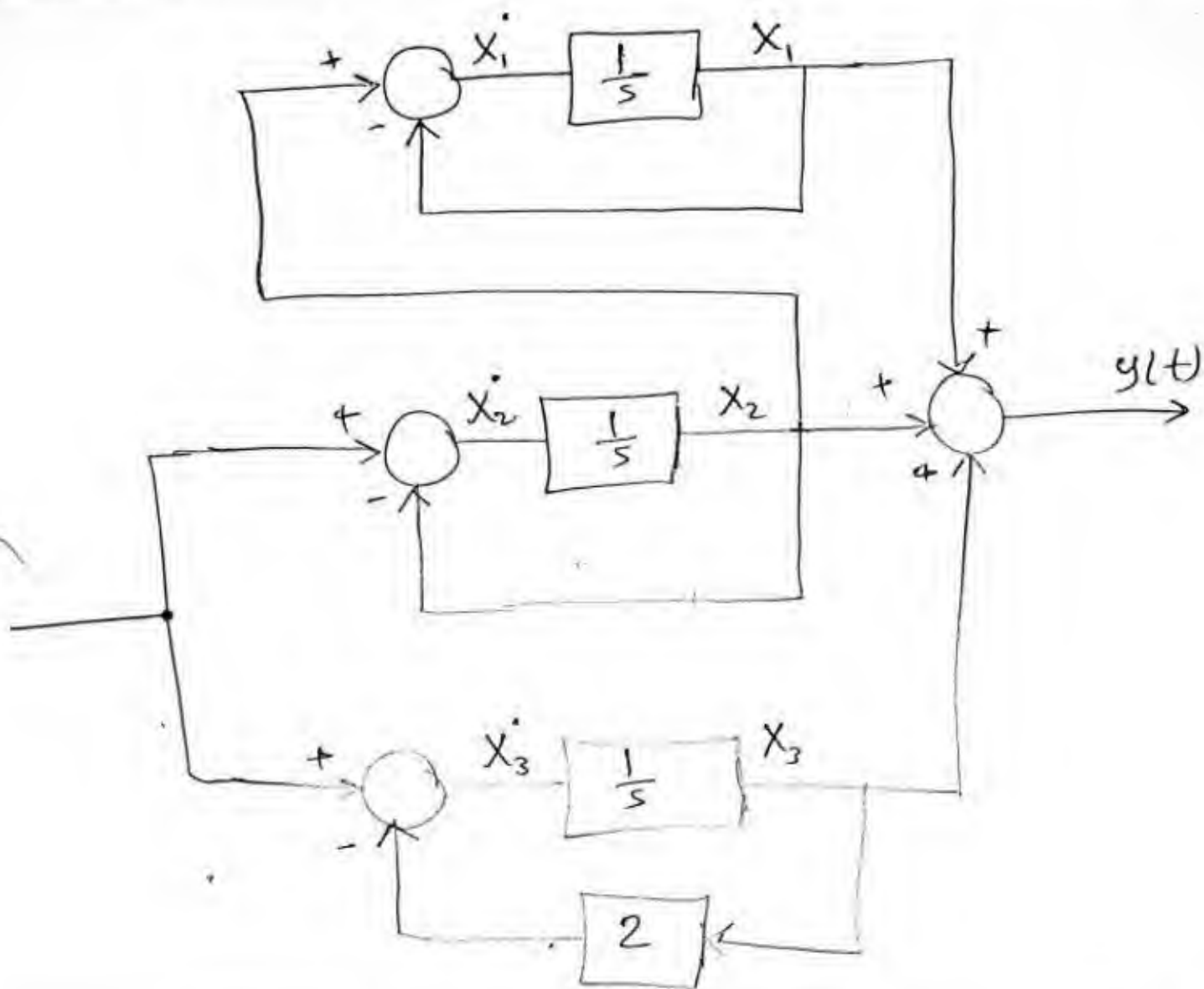
$$A_1 = \frac{2-6+5}{-1+2} = 1 \quad A_3 = \frac{8-12+5}{1} = 1$$

Put $s=0$ $\frac{5}{2} = A_1 + A_2 + \frac{A_3}{2}$

$$\hookrightarrow A_2 = 1$$

$$\dot{X}(t) = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} X(t)$$



state-space Analysis

Given the two state equations

$$\begin{cases} \dot{x} = Ax + Bu \\ y = cx \end{cases}$$

11 T.F

$$sX(s) = AX(s) + Bu(s) \rightarrow \textcircled{1}$$

$$Y(s) = CX(s) \rightarrow \textcircled{2}$$

$$sX(s) - AX(s) = Bu(s)$$

$$(sI - A)X(s) = Bu(s)$$

↓

sI

s - مفرقة في
هوية الوحدة

$I_{n \times n} \rightarrow$ identity matrix

$$X(s) = (sI - A)^{-1} Bu(s) \rightarrow \textcircled{3}$$

From $\textcircled{3}$ in $\textcircled{2}$

$$Y(s) = C(sI - A)^{-1} Bu(s)$$

$$T.F = \frac{Y(s)}{U(s)} = C(sI - A)^{-1} B$$

[2] ch. equation

$$|sI - A| = 0$$

→ نتائج معادله جذورها هي (Poles) النظام (system).

→ roots of ch. equation \equiv Poles \equiv eigen values.

[3] The system response to i/p $u(t)$
o/p in time domain.

if $x(0) \neq 0$

$$\dot{x}(t) = Ax(t) + Bu(t) \xrightarrow{L.T.}$$

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = x(0) + BU(s)$$

$$(sI - A)X(s) = x(0) + BU(s)$$

$$X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}BU(s)$$

Let: $\phi(s) = (sI - A)^{-1} \Rightarrow$ transition Matrix

$$X(s) = \phi(s) X(0) + \phi(s) B U(s)$$

$$y(t) = c x(t)$$

$$Y(s) = c X(s) = c [\phi(s) X(0) + \phi(s) B U(s)]$$

$$Y(s) \xrightarrow{\mathcal{L}^{-1}} y(t)$$

$$\boxed{\text{EX}} \quad \dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 \\ 2 \end{pmatrix} u(t)$$

$$y(t) = (0 \quad 1) x(t) \quad \& \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Find T.F & $y(t)$ for unit step response.

$\boxed{\text{Sol}}$

$$\text{T.F} = c(sI - A)^{-1} B$$

$$(sI - A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s(s+3) + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$*(sI - A)^{-1} = \frac{1}{s^2 + 3s + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$T.F = C (sI - A)^{-1} B$$

$$= \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{1}{s^2 + 3s + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{s^2 + 3s + 2} \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2s \end{pmatrix}$$

$$\boxed{T.F = \frac{2s}{s^2 + 3s + 2}}$$

ch-equation

$$|sI - A| = 0$$

$$s^2 + 3s + 2 = 0$$

$$X(s) = \phi(s) X(0) + \phi(s) B U(s)$$

$$\Phi(s) = (sI - A)^{-1} = \frac{1}{s^2 + 3s + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}$$

$$X(s) = \frac{1}{s^2 + 3s + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \dots$$

$$\frac{1}{s^2 + 3s + 2} \begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \frac{1}{s} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$\underbrace{\frac{1}{s}}_{U(s)}$
 Cause $u(t) = 1$

$$X(s) = \frac{1}{s^2 + 3s + 2} \left[\begin{pmatrix} 1 \\ s \end{pmatrix} + \begin{pmatrix} \frac{2}{s} \\ 2 \end{pmatrix} \right]$$

$$= \frac{1}{s^2 + 3s + 2} \begin{pmatrix} 1 + \frac{2}{s} \\ s+2 \end{pmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{pmatrix} \frac{s+2}{s} \\ s+2 \end{pmatrix}$$

[23]

$$X(s) = \begin{pmatrix} \frac{1}{s(s+1)} \\ \frac{1}{s+1} \end{pmatrix}$$

$$Y(s) = C \cdot X(s)$$

$$= (0 \quad 1) \begin{pmatrix} \frac{1}{s(s+1)} \\ \frac{1}{s+1} \end{pmatrix} = \frac{1}{s+1}$$

$$\underline{\underline{\mathcal{L}^{-1} \cdot \underline{\underline{I}}}}$$

$$y(t) = e^{-t}$$

(unit-step response)

[4] Controllability

The system is completely controllable if the system states can be changed by changing the system input.

* The ability of the control i/p signal of system to move any initial state to another final states during finite intervals ~~of~~ of time

$$x(t_0) \longrightarrow x(t_1)$$

* Controllability matrix (M_c)

$$M_c = (B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B)$$

. system is controllable

if $|M_c| \neq 0$, the system is Controllable.

2nd order

$$M_c = (B \quad AB)$$

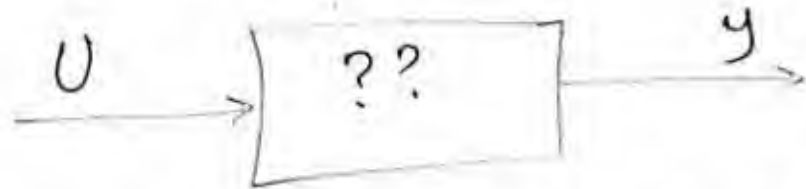
3rd order

$$M_c = (B \quad AB \quad A^2B)$$

$\boxed{2 \times 5}$

5 observability

لو عندك (states) مش عارف تقدر (estimation) تستخدم ال (observer)



لو عارف الخرج وعاليز تعرف ما بداخل النظام .
في الحالة دي نقول على النظام انه (observable)

* In some cases, the states couldn't be measured for the following reasons:

1- the location for physical states.

2- The measuring instruments are not valid.

→ in these case, estimation for these states is required.

* if the internal states of a system could be estimated (calculated) from the observation of o/p response (o/p), then system is called observable.

observability Matrix (M_o)

$$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

$|M_o| \neq 0$. observable

2nd order

$$M_o = \begin{pmatrix} C \\ CA \end{pmatrix}$$

3rd order

$$M_o = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$$

EX

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$C = (1 \quad 0 \quad 1)$$

check observability, controllability

$$M_c = (B \quad AB \quad A^2 B)$$

$$AB = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; A^2 B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$M_c = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

$$|M_c| = - \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

→ system is not controllable

$$\mathcal{M}_0 = \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix}$$

$$CA = (1 \quad 0 \quad 1) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = (1 \quad 2 \quad 1)$$

$$CA^2 = CA * A = (1 \quad 2 \quad 1) \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = (1 \quad 4 \quad -1)$$

$$\mathcal{M}_0 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 4 & -1 \end{pmatrix} \Rightarrow |\mathcal{M}_0| = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -6 + 2 = -4 \neq 0$$

→ system is observable.